346

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346

WORKING NOTES AND BIBLIOGRAPHY ON THE STUDY OF SHAPE IN HUMAN GEOGRAPHY AND PLANNING

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INTRODUCTION

Scientists and philosophers in many disciplines have long recognized the potential for gaining insights into processes by an examination of the forms generated by those processes. The seminal work for most fruitful recent studies has been Sir D'Arcy Wentworth Thompson's monumental accomplishment, On growth and form. This classic work has provided the stimulus for a continuing evaluation of the concept of "form" throughout science. Typical of such developments are the results of two symposia Essays on growth and form (edited by W. E. Le Gros Clark and P. B. Medawar, London, Oxford University Press, 1945) and Aspects of form: a symposium on form in nature and art (edited by L. L. Whyte, New York, American Elsevier Publishing Co., Inc., 1968).

Geographers have been among those who have attempted to formulate process-form arguments, but their work has often been impaired by the lack of any adequate means of describing shape.

Until recently, shape descriptions in human geography were overwhelmingly qualitative, with emphasis on visual representations, a situation which did little to facilitate comparative studies, or the generation of theory. Only in the physical branches of geography were endeavors made to construct quantitative measures of shape. In recent years the paucity of quantitative shape studies in human geography has been reduced by the advent of several diverse studies. Until now these studies have not been brought together in a single source. It is the goal of this bibliography to remedy this shortcoming.

For the purposes of this bibliography the term "shape" is interpreted as a set of properties possessed by any closed figure of at least two-dimensions which has a planar representation and which possesses precise boundaries. Thus, techniques which may be considered as means of describing the form of a surface (e.g., trend surface analysis) are not considered.

The emphasis in the bibliography is on quantitative studies in human geography; qualitative discussions of shape are only included if they are felt to be of special importance, or if they might form the basis of a quantitative investigation. Studies of purely topologic measures of shape (i.e., counts of the number of edges of a figure, counts of the number of contiguous figures, etc.) have also been excluded from the bibliography.

It has already been observed that quantitative shape studies have been undertaken in physical geography and related disciplines. Many of these studies involve shape indices

constructed using physical properties of the form (e.g., area, length of perimeter, etc.) and thus could also be applied to phenomena traditionally studied by human geographers (e.g., settlement configurations, political units, etc.) with little or no modification. For this reason such studies are included in this bibliography.[1] Further, as far as the authors are aware, the most advanced quantitative treatment of shape has occurred in psychology, in the description of randomly-generated shapes used in perceptual studies. Here multivariate techniques have been applied (see L. Zusne, Visual perception of form, New York, Academic Press, 1971). It is the authors' contention that such work could provide a valuable basis for multivariate shape studies in geography, and for this reason reference is made to the major psychological shape studies.

Because the sources listed in the bibliography are scattered, and many are difficult to obtain, each study which deals with the derivation of a shape index is described in considerable detail although no attempt is made to critically review any of the studies. Such review is currently being undertaken by the authors as part of their on-going research into the development of multivariate shape analysis in geography.

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[1] Many indices in physical geography are "variations on a theme" in that they are ratios constructed by selecting different combinations of several basic parameters of the shape. For this reason this bibliography presents a representative, rather than exhaustive, listing of such studies. Further, attention is devoted exclusively to two-dimensional indices. Thus, three-dimensional shape indices created in sedimentary petrology are absent from the bibliography.

Berry, B.J.L. and E. Neils (1972). Location, size and shape of cities as influenced by environmental factors: the urban environment writ large. In Perloff, H. S., editor, The Quality of the Urban Environment. Baltimore: Johns Hopkins Press, pp. 257-302.

Indicates how nationwide changes in the nature of urban structural processes are molding central place hierarchies more closely to the shape of the commuting fields of the United States metropolitan centers, irregardless of environmental differences.

Black, P. E. (1972). Watershed eccentricity. A contribution of the Watershed Model Studies Program. State University College of Forestry, Syracuse, New York.

A measure of basin shape (Γ) is derived by modifying the formula for the eccentricity of an ellipse. The modification allows the measure to become directional with respect to the mouth of the watershed and responsive primarily to the shape of the lower half of the drainage basin.

$$\Gamma = \frac{\sqrt{L_c^2 - W_L^2}}{W_L}$$

where: L_c = length from the basin mouth to its center of mass
W_L = width of basin perpendicular to L_c at the basin's center of mass

As $L_c \rightarrow W_L$, $\Gamma \rightarrow 0$ (i.e. lower values of Γ are associated with high peak potential). As L_c or W_L get large $\Gamma \rightarrow$ (low flood peaks).

Blair, D. J. and T. H. Biss (1967). The measurement of shape in geography: an appraisal of methods and techniques. University of Nottingham, Department of Geography, Bulletin of Quantitative Data for Geographers, 11.

The concept of shape is recognized as a set of properties -- compactness, elongation, and others which are spatially related and combine together. Compactness is considered the most important of these and the theoretical requirements of an index to measure this property are discussed. Existing measures are critically reviewed (Miller, 1953; Pounds, 1963; Cole, 1964; Boyce and Clark, 1964; Haggett, 1965; Bunge, 1966). Applies a new approach which involves imagining any area to be described to be split up into an infinite number of infinitesimal elements of area, and to derive an index based on the position of all these elements, using infinitesimal calculus.

Blair, D. J. and T. H. Biss (1967). -- Continued

Presents a new measure, the coefficient of compactness (C) of a shape, A, so that

$$C = \frac{D}{\sqrt{2\pi \int_A r^2 dxdy}}$$

where: D = area of shape to be measured
r = distance of a particular element of A from its centroid

Values range from 1.0 for a circle to 0.0 for increasingly uncompact figures. The index is used to measure the shape of political units ranging in size and status from English civil parishes to soverign states. Indicates how the principles developed for compactness may be extended to some related topics including elongation, orientation, centrality, and point distributions. Suggestions for the application of the approach to most branches of geography are outlined.

Boots, B. N. (1970). An approach to the study of patterns of cellular nets. Rutgers University, Department of Geography, Discussion Papers, 1.

Uses the Haggett (1965) shape index to measure the shape of individual cells within cellular nets (including river basins and English civil parishes). Using regression analysis, the relationships between shape, size, and contact number (a count of the number of cells contiguous to any individual cell) are examined.

Boots, B. N. (1971). The measurement of shape in geography. Unpublished paper, Rutgers University, Department of Geography. (Mimeographed).

Examines the requirements necessary for quantitative measures of shape. Proposes a four-fold classification of shape indices using two criteria. These criteria are, first, whether the index is a standard shape (SS) one or a non-standard shape (NS) one [SS indices are ones in which the difference between the shape of any individual and some standard shape (which can be either regular or irregular) is examined; NS indices do not facilitate direct comparison with another shape] and second, whether the index is a parametric (P) one or a non-parametric (NP) one [for P indices basic properties of the shape (area, perimeter, etc.) are used to compute the index; NP indices are free of such measures]. Thus, indices can be recognized as members of one of four classes (SSP, SSNP, NSP, NSNP). Examples of each class are reviewed; SSP (Haggett, 1965); SSNP (Lee and Sallee, 1970); NSP (Boyce and Clark, 1964); NSNP (Bunge, 1966; Taylor, 1970).

Boyce, R. R. and W.A.V. Clark (1963). Selected spatial variables and central district retail sales. Papers and Proceedings of the Regional Science Association, 11, 167-193.

Includes an examination of the relationship between CBD sales and urban configuration. Suggests that a circular shaped metropolis should be more advantageous for the total population to reach the CBD (and hence CBD sales will be higher) than an extremely elongated shape. Several other hypotheses are informally stated. Uses Boyce and Clark (1964) index (using both the location of the center of gravity and the CBD as central nodes) as a measure of shape. The simple correlation coefficient between the shape index and amount, or percentage of SMSA sales in the CBD is low; higher values are found for the shape index computed with respect to the location of the CBD ($r = -0.30$) than for the shape index computed with respect to the shape's center of gravity.

Boyce, R. R. and W.A.V. Clark (1964). The concept of shape in geography. Geographical Review, 54, 561-572.

Reviews the use of shape studies in human geography (found to be predominantly qualitative) and fluvial geomorphology and mineralogy (quantitative measures more prevalent). Extensive review of Bunge (1966). Presents new measure of shape based on the measurement of radial distances from a selected point within the figure to the circumference of the figure; thus

$$\sum \left| \frac{r_i}{\sum r_i} \cdot 100 - \frac{100}{n} \right|$$

where: r = radials extending outward from a central node
n = number of radials used

If the central node is the center of gravity of the shape the index for a circle is 0, with all other geometric forms having larger values (e.g., square = 12; cross = 18; star = 25) and forming a continuum with the value for a straight line being 175. Suggests this enables any figure to be ranked with respect to others and its closet pure geometric approximate determined. Shape curves are also generated by plotting the radial percentage numbers along a horizontal axis from 1 to $_n$, and the percentage distance for each radial along the vertical scale. Discusses extensions of their measure and use of it to evaluate the relationship between per cent SMSA sales in the CBD and shape of residential area, using site of CBD as nodal point; finds $r = 0.49$ (see Boyce and Clark, 1963).

Brown, D. R. and D. H. Owen (1967). The metrics of visual form: methodological dyspepsia. Psychological Bulletin, 68(4), 243-259.

In depth investigation into intercorrelations of shape variables, providing the basis for the factor space. Data relevant to the sampling characteristics of those variables on randomly-generated polygons are also examined. Eighty measures were subjected to Q-mode and R-mode principal axes factor analysis, with twelve factors resulting from the R-mode accounting for 87 per cent of the total communality. Measures include first four moments of area, perimeter, edge length, interior and exterior angles, radial lengths at one degree steps, and the ratio of major to minor axis.

Bunge, W. (1966). Theoretical geography. Revised edition. Lund Studies in Geography, Series C, General and Mathematical Geography, 1.

(Chapter 3, pp. 77-88)

Method is based on two theorems: (i) that any simple connected shape can be matched by a polygon of any number of sides, whose sides are of equal but variable length, (ii) that if the difference between all vertices of the polygon are summed in a standard manner there exists just one set of sums that uniquely describe the polygon shape. The approach is as follows. First, the original shape is approximated by an equal-sided polygon of n sides. Then a series of lag measures are made for each vertex; lag 1, lag 2, lag 3, etc. For lag 1 measurements every other vertex is skipped (i.e., the distance between the first and third vertex, second and fourth vertex, etc., is measured). These measures are summed to give a shape sum (SS_1) and squared and summed to give SS_1^2. For lag 2 measures two vertices are skipped (first and fourth), second and fifth, etc.) and these are summed to give SS_2 and squared and summed to give SS_2^2. The lagging procedure is continued until all previously determined shape sums reappear, indicating all unique sums have been determined. For a polygon of n sides there are (n - 2) such unique sums, which taken together constitute the shape measure. Shapes may be measured in terms of the length of polygon side and thus different shapes compared. The measure is evaluated for four geometric shapes and 97 Mexican communities.

Chorley, R. J. (1959). The shape of drumlins. Journal of Glaciology, 3, 339-344.

The author illustrates the usefulness of applying the lemniscate standard (Chorley et al, 1957) to drumlins, and proposes that the standard may be useful in the investigation of the mechanics of drumlin formation.

Chorley, R. J., D. E. G. Malm and H. A. Pogorzelski (1957). A new standard for estimating drainage basin shape. American Journal of Science, 255, 138-141.

Authors suggest, for drainage basin analysis, that the lemniscate loop be substituted for the circle as a standard of comparison. In this way they replace the circularity measure of Miller (1953) with their own ratio:

$$L.R. = \frac{P}{P_m}$$

where: L.R. = lemniscate ratio
P = perimeter of the ideal lemniscate shape
P_m = perimeter of the measured drainage basin shape

Chow, Ven Te, Editor (1964). Handbook of applied hydrology: a compendium of water-resources technology. New York: McGraw-Hill.

(pp. 51-52)

Reviews Horton's (1932) form ratio, inverted form ratio used by Corps of Engineers ("Unit Hydrograph Compilations", U. S. Corps of Engineers, Department of the Army, Washington District, Civil Works Invest., Project CW 153; three volumes 1949; one volume 1954), Miller's (1953) circularity ratio, Schumm's (1956) elongation ratio, and Chorley et al's (1957) lemniscate ratio.

Cole, J. O. (1964). Study of major and minor civil divisions in political geography. Paper presented at the 20th International Geographical Congress. Sheffield, England, July - August 1964.

Proposes a coefficient of compactness (I).

$$I = \frac{A}{A_x} .100$$

where: A = area of figure
A_x = area of smallest circle to enclose figure

Values extend from 100 (a circle) to 0 as a figure becomes increasingly uncompact (cf., Gibbs, 1961 and Haggett, 1965).

Domanski, R. (1967). Comments on the paper by Nystuen. Papers of the Peace Research Society, 7, 129-131.

Notes that although Nystuen (1967) introduces the concept of an -neighborhood he does not adequately define or interpret it. Also suggests boundaries can be defined topologically as well as functionally. Illustrates several topological properties of boundary segments.

Fairthorne, D. (1965). The distances between pairs of points in towns of simple geometric shapes. In Almond, J., editor, Proceedings of the second international symposium on the theory of road traffic flow. London.

(pp. 391-406)

Compares different geometric city shapes in terms of their general accessibility. Illustrates that compactness of city shape is directly related to overall accessibility.

Gibbs, J. P. (1961). A method for comparing the spatial shapes of urban units. In Gibbs, J. P., editor, Urban research methods. Princeton, New Jersey: Van Nostrand Co., Inc.

(pp. 99-106).

Measures the degree to which the shape of any city has a circular form. Uses formula

$$M_c = \frac{100\ A_a}{(3.1416)(D_p/2)^2}$$

where: M_c = measure of circularity in shape
A_a = actual area contained in the urban unit
D_p = distance between the two most distant points on the boundary of the unit

Range of values: 100.00% (completely circular city) to 0.00% (completely linear city). Illustrates the effect of the choice of boundary (political, urbanized, urbanized area excluding non-contiguous parts) on the resulting shape index for a city.
(cf. Cole, 1964 and Haggett, 1965)

Haggett, P. (1965). Locational analysis in human geography. London: Edward Arnold Ltd.

(pp. 50-51)

The Gibbs (1961) shape index is restated as

$$S = (1.27A)/L^2$$

where: S = shape index
A = area of figure
L = long axis of figure

A sample of 100 counties (municipies) drawn at random from Brazilian data are evaluated using the index. It is stated that the values of S for triangular, square, and hexagonal lattices are 0.42, 0.64, and 0.83, respectively. It is proposed that these values can be used to divide a distribution into three zones, corresponding to the three lattices.

Haggett, P. (1965). -- Continued

(pp. 227-229)

Descriptions of shape are discussed and the work of Gibbs (1961) and Bunge (1966) is reviewed.
(cf. Gibbs, 1961 and Cole, 1964)

Haggett, P. and R. J. Chorley (1969). Network analysis in geography. London: Edward Arnold.

(pp. 57-74)

Discusses the problems involved in measuring both the shape of open links (lines, including river systems and individual transport links) and closed links (figures of two or more dimensions). In particular, the Steinhaus paradox and its implications for the measurement of open links is discussed. For closed links a number of measures are reviewed briefly (Horton, 1932; Miller, 1953; Schumm, 1956; Boyce and Clark, 1964; Haggett, 1965; Stoddart, 1965; Bunge, 1966; Blair and Biss, 1967). Topologic measures of overall network shape are also discussed briefly.

Hartman, G. W. (1950). The central business district - a study in urban geography. Economic Geography, 26, 237-244.

Discusses the shape of central business districts as variations from basic geometric forms such as the star, the diamond, and the cross. Suggests distinct geometric patterns are often recognizable.

Horton, R. E. (1932). Drainage basin characteristics. Transactions of the American Geophysical Union, 13, 350 - 361.

Introduces his "form ratio" or "width-length ratio".

$$F = \frac{A}{L^2}$$

where: F = form ratio
A = area of basin
L = longest length (major axis) in basin

Briefly reviews another previously-developed measure (C).

$$C = P/(2\sqrt{\pi A})$$

where: P = perimeter of the drainage basin
A = area of the drainage basin

Hudson, J. C. and P. M. Fowler (1966). The concept of pattern in geography. University of Iowa, Department of Geography, Discussion Paper Series, 1.

Notes the distinction between pattern and shape in terms of the dimensions of the objects being studied.

Kulldorf, G. (1955). Migration probabilities. Lund Studies in Geography, Series B, Human Geography, 14.

Considers relationships between migration and size and shape of areas involved, based on the assumption shape of areas will be either circular, rectangular, or triangular. Practically identical migration probabilities are obtained irregardless of shape assumed.

Lee, D. R. and G. T. Sallee (1970). A method of measuring shape. Geographical Review, 60(4), 555-563.

Critically reviews Boyce and Clark (1964) and Bunge (1966). Suggests a new approach employing the "symmetric difference metric" in which "instead of attempting to assign each shape a unique number, a comparison is made between two shapes, one of which is unknown (for example, a drainage basin, a village or a central business district), and the other easily described and conceptualized in non-technical terms (for example, a circle, a square, or a triangle)." The index (Γ) is given by

$$\Gamma = 1 - \text{area}\ (K \cap L)/\text{area}(K \cup L)$$

where: K = unknown shape
L = standard shape

In determing ($K \cap L$) and ($K \cup L$), ($K \triangle L$) (the area of non-fit between K and L) is minimized. L can be varied as desired, and may be regular or non-regular; increasing fit between K and L is indicated by the approach of Γ to zero. The method has the advantages that the size and scale of K is of no importance and that there is no problem in handling punctured or fragmented shapes which plague most other indices. Γ is evaluated for 25 Sudanese villages.

Luu-Mau-Thanh (1962). Distributions théoriques des distances entre deux points répartis uniformément sur une surface. In Sutter, J., editor, Les déplacements humains. Aspects méthodologiques de leur mesure. (Human displacements. Measurement methodological aspects). Entretiens de Monaco en sciences humaines (première session 1962, 24-29 Mai). Paris, Hachette.

(pp. 173-184)

Produces frequency distributions of the distance between points chosen at random from a uniform distribution of points within a circle, an equilateral triangle, and a square.

Marsden, B. S. (1970). Urban delimitation on a density basis: Brisbane, 1861-1966. New Zealand Geographer, 26(2), 151-161.

Uses area, perimeter, longest axis, largest inscribing circle, smallest circumscribing circle, shape index of Gibbs (1961) and an orientation parameter in time-series analysis to characterise Brisbane's growth.

Massam, B. H. (1970). A note on shape. Professional Geographer, 22(4), 197-199.

Discusses an approach to administrative areas by examining measures of the shape and functional effiency of each area in an attempt to link the form of the area to the process which operates within an area. Rural Operating Areas of the Ontario Hydro-Electric Power Commission are examined using Haggett's (1965) index as a measure of shape, and number of employees per customer as a measure of economic efficiency.

Massam, B. H. (1972). The spatial structure of administrative systems. Association of American Geographers, Commission on College Geography, Resource Papers, 12.

Briefly reviews the work on shape of Haggett (1965), Bunge (1966) and Lee and Sallee (1970). Restates the derivation and use of a new shape measure,

$$S_v = A^2/2 \quad I_a$$

(see Massam and Burghardt, 1968 and Massam and Goodchild, 1971)

Massam, B. H. and A. F. Burghardt (1968). The administrative subdivisions of southern Ontario: an attempt at evaluation. Canadian Geographer, 12(3), 125-134.

Introduces a measure of compactness and a measure of efficiency for administrative areas, both of which depend on the moment of inertia of population about an administrative center

$$I = \sum_{i=1}^{n} m_i r_i^2$$

where: I = moment of inertia
m_i = number of people at the central point in township i
r_i = distance of m_i from the administrative center
n = the number of townships in the administrative area

and

Massam, B. H. and A. F. Burghardt (1968). -- Continued

$$E = \frac{I_g}{I_x}$$

where: E = efficiency
I_g = moment of inertia about center of gravity or centroid of administrative area
I_x = moment of inertia of administrative area

(see also Massam, 1972 and Massam and Goodchild, 1971)

Massam, B. H. and M. F. Goodchild (1971). Temporal trends in the spatial organization of a service agency. Canadian Geographer, 15(3), 193-206.

Two measures of the degree of geometric optimality of an area are derived using properties of moments of inertia. The moment of inertia (I) is defined for any point x in an area as

$$I_x = \int_a r^2 \, da$$

(i.e., the sum over the area of each minute segment of area multiplied by the square of the distance (r) separating it from the point x). Properties of I are used to derive two measures. For any given area, I_x is a minimum for a point located at the center of gravity of the area. An efficiency measure (E) is defined as

$$E = I_a/I_x$$

where: I_a = moment of inertia at the center of gravity of the area

Values range from 1.0 (when x is centrally located) to 0.0 (for a peripheral location). A second property of I enables I_a to be calculated for any shape of area A; I_a will be a minimum when the shape is a circle. A shape index (S) is defined

$$S = A^2/2\pi \, I_a$$

(i.e., I of a circle of area A at its center of gravity divided by I of the actual shape at its center of gravity). Values range from 1.0 for a circle to 0.0 for an infinitely extended strip. Both E and S are independent of scale (cf., Blair and Biss, 1967). These measures are used in an analysis of temporal changes in the spatial organization of Rural Operating Areas of the Ontario Hydro-Electric Power Commission (see Massam, 1970). A tendency for the shape of each area to approach optimum compactness is identified.
(see also Massam and Burghardt, 1968 and Massam, 1972)

Medawar, P. B. (1945). Size, shape, and age. In Le Gros Clark, W. E. and P. B. Medawar, editors, Essays on growth and form: presented to D'Arcy Wentworth Thompson. London: Oxford University Press.

Through very complete examples the author explains how Thompson's "method of transformations" may be used to advantage in growth analysis. Through mathematically specifying the direction of shape changes he develops the method to the point where rates of change of shape become understandable and useful.

Miller, V. C. (1953). A quantitative geomorphic study of drainage basin characteristics in the Clinch Mountain area, Virginia and Tennessee. Columbia University, Department of Geology, Contract N6, ONR, Project NR 389-042, Technical Report, 3.

Introduces a circularity ratio (R_c) which compares the area of a drainage basin to the area of a circle having the same perimeter:

$$R_c = \frac{A}{(P/2\pi)^2} \left[= \frac{4A}{P^2} \right]$$

where: A = area of basin
P = perimeter of basin

Morisawa, M. E. (1958). Measurement of drainage-basin outline form. Journal of Geology, 66, 587-591.

Tests the effectiveness of Horton's (1932) form ratio, Miller's (1953) circularity ratio, Schumm's (1956) elongation ratio, and Chorley et al's (1957) lemniscate ratio as factors in the hydrology of a watershed. Elongation ratio (R_e) and circularity ratio (R_c) provided the only statistically significant regression coefficients. Author concludes that other factors must be responsible for hydrologic characteristics of basins.

Morisawa, M. E. (1968). Streams: their dynamics and morphology. New York: McGraw-Hill.

The measures of Horton (1932), Miller (1953), Schumm (1956), Chorley et al (1957) and the Corps of Engineers ratio which is the inverse of Horton's form ratio ("Unit Hydrograph Compilations", U. S. Corps of Engineers, Department of the Army, Washington District, Civil Works Invest., Project CW 153: three volumes 1949; one volume 1954) are tabulated).

Nystuen, J. D. (1965). Effects of boundary shape and the concept of local convexity. Paper presented at the Regional Science Association Annual Meeting, Philadelphia, Pennsylvania, November 1965. (Mimeographed).

Introduces a concept (local convexity) useful in analyzing the interplay between boundaries and spatial processes. Emphasis is on the shape of line (one-dimensional) boundaries and their relation to the intensity of spatial processes. Argues that in order to analyze the effectiveness of boundary shape in this respect measures of boundary length and curvature are necessary. Problems in the measurement of the former are discssed and the related concepts of epsilon ( ) length (see Perkal, 1966), - radius, and - neighborhood are reviewed and extended. Functional definitions of - neighborhoods are discussed.

Nystuen, J. D. (1966). Effects of boundary shape and the concept of local convexity. Michigan Inter-University Community of Mathematical Geographers, Discussion Papers, 10A.

See Nystuen (1965).

Nystuen, J. D. (1967). Boundary shapes and boundary problems. Papers of the Peace Research Society, 7, 107-128.

Refinement and extension of Nystuen (1965, 1966). (see also Domandski, 1967)

Olsson, G. (1965). Distance and human interaction: a review and bibliography. Regional Science Research Institute, Bibliography Series, 2.

Brief reviews of Bunge (1966) and Gibbs (1961).

Ongley, E. D. (1970). Drainage basin axial and shape parameters from moment measures. Canadian Geographer, 14(1), 38-44.

Direct measures of basin shape and symmetry are accomplished by reduction of the basin network to a simple frequency distribution which is described using standard statistical techniques (moment measures). Suggests the property of shape involves the concepts of elongation (a function of both standard deviation and kurtosis) and degree of symmetry (measured by skewness). Although the analysis is restricted to drainage basins the approach could be applied to any closed links containing a connected set of open links and a distinct node (e.g., a regional road network).

Pedersen, P. O. (1967). On the geometry of administrative areas. Copenhagen, M.S. Report.

Replicates the Haggett (1965) study of Brazilian municipios for Danish communes. Concludes that administrative net in Denmark is basically triangular (about 60 per cent) with hexagonal elements largely absent.

Perkal, J. (1966). On the length of empirical curves and an attempt at objective generalization. Michigan Inter-University Community of Mathematical Geographers, Discussion Papers, 108.

Suggests that the measurement of the length of a line must involve a consideration of the effect of scale. Proposes a standard measure, epsilon length (L_ε), to facilitate this.

$$L_\varepsilon(x) = \frac{a_\varepsilon(x) - \pi\varepsilon^2}{2\varepsilon}$$

where: $L_\varepsilon(x)$ = epsilon length of an empirical line, x
ε = fixed small distance
$a_\varepsilon(x)$ = area within ε distance from the line

If the line is closed the term $\pi\varepsilon^2$ (representing the two semi-circles at each end of the line) is ignored. $L_\varepsilon(x)$ is used by Nystuen (1965, 1966, 1967) in his consideration of the effects of boundary shape on boundary processes.

Polosa, A. I. and A. M. Smirnov (1970). Study of the shape of rural communities. (From Vestnik Moskovskogo Universiteta Geografika, 1969, No. 6, 48-51). Soviet Geography: Review and Translation, 11(8), 645-649.

Shape, size, and spatial orientation are viewed as representing a unity which reflects the interaction of two opposed tendencies: (i) minimum-distance or minimum-energy concepts (so called "efficiency" concepts) and (ii) the need, at certain times, for expenditure of energy, regardless of its magnitude. To study this phenomenon the authors propose a measure in which they evaluate the metric difference between the study area and a circle of equal area with its center at the centroid of the study area (cf., Lee and Sallee, 1970). They also include their dispersion measure (K) which compares the perimeter of the study area (P) with that of the circle (C) so that

$$K = P/C$$

Pounds, N.J.G. (1963). Political geography. New York: McGraw-Hill.

(p. 46)

The boundary length of a figure in relation to its area is proposed as a measure of shape.

Reed, B., C. J. Galvin, Jr. and J. P. Miller (1962). Some aspects of drumlin geometry. American Journal of Science, 260, 200-210.

Authors measure form, orientation and spacing of selected drumlins. Find that individual drumlins departed only slightly from the ideal ellipsoidal form with which they were compared.

Reock, E. C., Jr. (1961). A note: measuring compactness as a requirement of legislative apportionment. Midwest Journal of Political Science, 5(1), 70-74.

Suggests the use of a compactness measure to aid in apportioning legislators. Measure suggested compares legislative districts with the smallest possible circumscribing circle. (See Cole, 1964)

Richardson, L. F. (1961). The problem of contiguity. An appendix to Statistics of deadly quarrels, edited by Wright, Q. and C. C. Lienau. Pittsburgh: Boxwood Press. Reprin in General Systems, Yearbook of the Society of General Systems, 6, 139-187.

Introduces a measure of compactness (H) (cf., Horton, 1932):

$$H = \frac{2(\pi A)^{\frac{1}{2}}}{P}$$

where: H = homoplaty or compactness
A = area of the closed figure
P = perimeter of the closed figure

Also utilizes a measure of radial moments of area (C) (cf., Wilkins and Shaw, 1971):

$$C \quad \frac{1 \quad M_o^2}{2\pi \; M_2}$$

where: M_o = area of the whole figure
M_2 = second moment of area of figure

Both H and C are independent of size and are evaluated for several regular geometric figures. Data is provided for the measures for an extensive list of countries.

Rydell, C. P. and B. H. Stevens (1968). Air pollution and the shape of urban areas. Journal of the American Institute of Planners, 34(1), 50-51.

Consideration of urban forms which will minimize effects of automobile emissions. Optimal urban form minimizes combined costs of total vehicle-miles of automobile traffic and total vehicle-generated air pollution.

Schumm, S. A. (1956). Evolution of drainage systems and slopes in badlands at Perth Amboy, New Jersey. Bulletin of the Geological Society of America, 67, 597-646.

Introduces his elongation ratio (R_e) (ratio between the diameter of a circle having the same area and the maximum length of the basin) as a restatement of the Wadell sphericity ratio used in petrology (see Krumbein, W. C. and F. J. Pettijohn, Manual of sedimentary petrography, New York, Appleton-Century Co., 1938, p. 284). Computes the ratio for selected drainage basins and plots elongation against relief stage. Concludes drainage basin shape varies with relative relief.

$$R_e = \frac{d}{L}$$

where: d = diameter of a circle of same area
L = longest length (major axis) of area

Smart, J. S. and A. J. Surkan (1967). The relation between mainstream length and area in drainage basins. Water Resources Research, 3(4), 963-974.

Uses Schumm's (1956) elongation ratio and a shape measure (S) which is given by

$$S = L_o/A^{\frac{1}{2}}$$

where: L_o = some characteristic length of the basin
A = area of the basin

Compares the measures of Miller (1953), Schumm (1956), and Chorley et alia (1957) with respect to their data.

Stenson, H. H. (1966). The physical factor structure of random forms and their judged complexity. Perception and Psychophysics, 1, 303-310.

Study relates the perceived complexity of 20 random forms to 24 physical variables of which the most important are: number of turns, length of perimeter, squared-perimeter to area ratio, and the variance of the interior angles of the form. The list of 24 variables is fairly exhaustive and also includes proportion of arcs to total number of turns, area, proportion of straight perimeter, orientation, elongation index (similar to a/b in Werrity, 1969), areal

Stenson, H. H. (1966). -- Continued

distribution index (unrefined version of Blair and Biss, 1967), variance index (cf., Taylor, 1970), mean, variance, and skewness of straight lengths, arc lengths and internal angles. Author provides a correlation coefficient matrix of all variables on each other and performs a principal axes factor analysis with rotations using varimax criterion. He also provides an in-depth discussion of the resultant factor loadings and factor characteristics.

Stoddart, D. R. (1965). The shape of atolls. Marine Geology, 3, 369-383.

The shapes of 99 atolls are measured using six measures of shape. These are three geomorphic form ratios (Horton, 1932; Miller, 1953; Schumm, 1956), the measures of Boyce, and Clark (1964) and Bunge (1966), and a new measure, the ellipticity index (I_c), derived by the author.

$$I_c \quad \frac{2a}{2b} \text{ or } \frac{L_1}{2b}$$

where: $b = \dfrac{A_1}{(L_1/2)\pi}$

A_1 = area of figure
L_1 = long axis of figure

(i.e., the index compares the unknown shape with an ellipse of the same area and longest axis). All six measures are reviewed with particular reference being paid to their ability to differentiate between atolls.

Taylor, P. J. (1970). Distances within shapes: an introduction to a family of finite frequency distributions. University of Iowa, Department of Geography, Discussion Paper Series, 16.

Shows how the distribution of distances within a shape can be used to measure several spatial characteristics of the shape. The approach is to reduce a two-dimensional shape into a one-dimensional frequency distribution. To operationalize the approach any shape, if overlaid by a grid, can be considered an array of ones and zeros, with the former representing grid cells within the shape, and the latter cells outside the shape. Distance distributions are generated by taking each 1-cell in turn and calculating the distance (in units of cell side) to every other 1-cell. These distances are then aggregated and the resulting distribution is examined using moment measures. These measures are then interpreted. The advantage of the approach appears to be its ability to handle very irregular or fragmented shapes. A set of

Taylor, P. J. (1970). -- Continued

geometric shapes and four "real world" shapes are evaluated using the measure. In addition, the approach is used to determine the effects of spatial structure (including shape) on migration patterns in mid-nineteenth century Liverpool.

Taylor, P. J. (1971). Distances within shapes: an introduction to a family of finite frequency distributions. Geografiska Annaler, Series B -- Human Geography, 53B(1), 40-53.

See Taylor (1970).

Thompson, D'Arcy W. (1942). On growth and form. Cambridge: Cambridge University Press.

(Original edition published 1917; 1942 edition reprinted and published as two volumes 1961)

Pioneering, and now classic, work on the relation of process to form, particularly in the biological sciences. Topics pertinent to shape analysis include a discussion of cell partitioning processes, space partitioning processes, conditions associated with hexagonal symmetry, and the theory of transformations.

Trenhaile, A. S. (1971). Drumlins: their distribution, orientation, and morphology. Canadian Geographer, 15(2), 113-126.

One hundred drumlins from 20 locations in southern Ontario were studied. The measures of Chorley et alia (1957) and Reed et alia (1962) are compared for drumlin shapes.

Werrity, A. (1969). On the form of drainage basins. Pennsylvania State University, Department of Geography, Papers in Geography, 1.

In his search for a conceptual model for the measurement of form the author reviews the problem of scale in shape measurement. He uses Miller's (1953) circularity ratio and Schumm's (1956) elongation ratio, as well as another elongation ratio:

$$a/b$$

where: a = length of the major axis of the drainage basin
b = length of the minor axis of the drainage basin

From a factor analytic study he concludes "Size and shape are inseparable for describing physical objects and the manner in which they are arranged in space."

White, C. L. and G. T. Renner (1957). College geography: natural environment and human society. New York: Appleton-Century Crofts.

(pp. 590-599)

Propose five-fold qualitative classification of the shapes of countries. The classes are: compact (e.g. Rumania), attenuated or elognated (Chile), prorupted (U.S.S.R.), perforated or punctured (South Africa) and fragmented (Greece).

Wilkins, C. A. and J. Shaw (1971). Measures of shape distortion in urban geography. Australian Geographer, 11(6), 593-595.

Authors review the shape measures of Boyce and Clark (1964) and Simmons (J. W. Simmons, "Relationships between the population density pattern and site of cities," unpublished M.A. thesis, University of Chicago, 1962), noting that both may be used as measures of shape for cities insofar as they are functions of the center and boundary of cities. Two indices are introduced which are measures of the distribution of population within a city. The first utilizes polar coordinates to lead to a statement of the city's deviation from the symmetrical distribution of its population:

$$D = \int_0^{2\pi}\int_0^{\infty} d^2(r, \Theta) r \, dr \, d\Theta - (1/2\pi)\int_0^{\infty}\left[\int_0^{2\pi} d(r, \Theta) d\Theta\right]^2 r \, dr$$

where: D = deviation from symmetry
r, Θ = polar coordinates with the city center as pole
$d(r, \Theta)$ = population density at point (r, Θ)

and the city is assumed to extend to infinity in all directions. The second uses the chi-squared statistic to compare the observed population distribution with that expected in a circularly symmetric city:

$$\chi^2 = \sum_{i,j} (P_{ij} - T_{ij})^2 / T_{ij}$$

where: P_{ij} = observed population of i-jth areas
T_{ij} = theoretical population

P_{ij} is obtained by applying a polar coordinate grid of non-specified size over the city with its center as the pole and then computing by use of:

$$P_{ij} = \frac{\Theta j + 1 - \Theta j}{2\pi} \sum_{j=1}^{n} P_{ij}$$

The statistic is distributed approximately as chi-squared with (m X n) - m degrees of freedom.

Willis, K. G. (1972). The influence of spatial structure and socio-economic factors on migration rates. A case study: Tyneside, 1961-1966. Regional Studies, 6, 69-82.

Uses the Blair and Biss (1967) shape measure in conjunction with other urban parameters in order to investigate influence of these variables on migration rates. Concludes shape has a negligible influence.

Woodruff, J. F. (1964). A comparative analysis of selected drainage basins. Professional Geographer, 16(4), 15-19.

Uses length, areal and elongation measures, as well as others in a search for quantitative expressions of morphological differences in drainage basins.

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